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I.Sc. Class-XI, Date- 21-04-2020
Permutations and Combinations.
Miscellaneous Exercise

Q.1. How many words, with or without meaning each of 2 vowels and 3 consonants can be formed from the letters of the word 'DAUGHTER'?

So. We have formed from the letters of the word Daughter.

Number of vowel = A, E, U = 3

Number of consonants = D, G, H, T, R = 5

2 vowels out of 3 vowels can be selected = ${}^3C_2$ ways

3 consonants out of 5 consonants can be selected = ${}^5C_3$ ways

And 2 vowels and 3 consonants can be ~~selected~~ arranged in $\lfloor 5$ ways

Hence, by Fundamental Principle of Counting, total number of ways $= {}^3C_2 \times {}^5C_3 \times \lfloor 5 = \frac{\lfloor 3}{\lfloor 2 \lfloor 3-2} \times \frac{\lfloor 5}{\lfloor 3 \lfloor 5-3} \times \lfloor 5$

$= \frac{\lfloor 3}{\lfloor 2 \lfloor 1} \times \frac{\lfloor 5}{\lfloor 3 \lfloor 2} \times \lfloor 5 = \frac{3 \lfloor 2}{\lfloor 2} \times \frac{5 \times 4 \times \lfloor 3}{\lfloor 3 \times 2 \times 1} \times 5 \times 4 \times 3 \times 2 \times 1$

$= 3 \times 10 \times 120 = 3600$ ways.

$[\lfloor 1 = 1$, $\lfloor 5 = 5 \times 4 \times 3 \times 2 \times 1 = 120]$

Q.2. How many words, with or without meaning, can be formed using all the letters of the word 'EQUATION' at a time so that the vowels and consonants occur together?

Soln. Given word is EQUATION.

Number of vowels = E, U, A, I, O = 5

Number of consonants = T, Q, N = 3

These vowels and consonants can be arranged in $\lfloor 2$ ways.

Hence, by FPC, total number of ways

$= \lfloor 5 \times \lfloor 3 \times \lfloor 2 = 5 \times 4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1 \times 2 \times 1 = 1440$ Ans

Miscellaneous Exercise

Q-3. A Committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of
(I) exactly 3 girls? (II) atleast 3 girls?

Soln (I) Out of 7 persons, we want to select exactly 3 girls
i.e. remaining 4 persons will be boys.
4 boys out of 9 boys can be selected in $^9C_4$ ways.
3 girls out of 4 girls can be selected in $^4C_3$ ways.
Hence, by Fundamental Principle of Counting, total number of ways for making the committee

$= {}^9C_4 \times {}^4C_3 = \frac{\lfloor 9}{\lfloor 4 \lfloor 9-4} \times \frac{\lfloor 4}{\lfloor 3 \lfloor 4-3}$ $\left[\because {}^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}\right]$

$= \frac{9\times 8\times 7\times 6\times \lfloor 5}{4\times 3\times 2\times 1\times \lfloor 5} \times \frac{4\lfloor 3}{\lfloor 3\times \lfloor 1} = 9\times 7\times 2\times \frac{4}{2\times 1} = 9\times 7\times 8 = 504$ ways.

(II) We have to select atleast 3 girls.
In first case. 4 boys out of 9 boys can be selected in $^9C_4$ ways
And 3 girls out of 4 girls can be selected in $^4C_3$ ways.
Hence by Fundamental Principle of Counting, total number of ways in this case $= {}^9C_4 \times {}^4C_3 = \frac{\lfloor 9}{\lfloor 4 \lfloor 9-4} \times \frac{\lfloor 4}{\lfloor 3 \lfloor 4-3}$

$= \frac{\lfloor 9}{\lfloor 4 \lfloor 5} \times \frac{\lfloor 4}{\lfloor 3 \lfloor 1}$

$= \frac{9\times 8\times 7\times 6\times \lfloor 5}{4\times 3\times 2\times 1\times \lfloor 5} \times \frac{4\times \lfloor 3}{\lfloor 3}$

$= 9\times 2\times 7\times 4 = 504$ ways.

In second case, 3 boys out of 9 boys can be select in $^9C_3$ ways
And, 4 girls out of 4 girls can be select in $^4C_4$ ways.
Hence, by Fundamental Principle of Counting, total number of ways in this case $= {}^9C_3 \times {}^4C_4 = \frac{9\times 8\times 7\times \lfloor 6}{\lfloor 3\times \lfloor 9-3} \times \frac{\lfloor 4}{\lfloor 4 \lfloor 4-4}$

$= \frac{9\times 8\times 7\times \lfloor 6}{3\times 2\times 1\times \lfloor 6} \times \frac{1}{\lfloor 0} = 84.$

Hence, by fundamental Principle of Counting, total number of ways
$= 504 + 84 = 588$ Ans.

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B.Sc. part-I (H), paper-I
Set Theory Date- 21-04-2020.

Q.1 Define partial order, linear order and well order on a set with an example of each one of them.

Ans: Partial Order: →

Let X is any non-empty set and R is a relation in X, then R is called a partial order relation in X when R is antisymmetric and transitive. If we write R as $\leq$, then '$\leq$' is a partial order if:

(i) For every $x, y \in X$, if $x \leq y$ and $y \leq x$ then $x = y$ [anti-symmetry]

(ii) For every, $x, y, z \in X$, if $x \leq y$ and $y \leq z$ then $x \leq z$ [transitivity]

The set $(X, \leq)$ is then called a partially ordered set, we shall take $x \leq y$ and $y \geq x$ as synonymous. For simplicity $(X, \leq)$ is written as X.

For any set X, the inclusion relation $\subseteq$ is a partial order in $P(X)$.

Linear order: → If for every $x, y \in X$, $x \leq y$ or $y \leq x$ then $\leq$ is said to be ~~total~~ linear order relation in X.

For example: In the set $X = \{1, 2, 3\}$, $A = \{1, 2\} \in P(X)$ and $B = \{2, 3\} \in P(X)$, but neither $A \subseteq B$ nor $B \subseteq A$.

~~Well ord~~ In the set $P(X)$, $\subseteq$ is a linear order.

Set Theory B.Sc. part – I(H), paper – I, 21-04-2020.

Rest part of Q.1.

Well order: → Let R is a relation in X, then R is called a well-ordering which for each subset A of X has a unique R-first element. Then X is called R-well-ordered.

When R is a partial order then the uniqueness is obtained from antisymmetry.

Let $(X, \leq)$ be any partially ordered set and X has an element a such that for each x in X, $a \leq x$ then a is called the first or least element of X.

Thus a is called the first element of X if and only if a is comparable with all other elements of X and no element of X is a predecessor of a.

Let $(X, \leq)$ be a partially ordered set and $A \subseteq X$. Then $l (\in X)$ is called a lowerbound of A if $l \leq x$ is true for each element x of A.

An element $u (\in X)$ is called an upper bound of $A (\subseteq X)$ if $x \leq u$ is true for each member of x of A.

Let $(X, \leq)$ be any partially ordered set and X has an element a such that for each x in X, $a \leq x$ then a is called the first or least element of X.

Thus a is called the first element of X if and only if a is comparable with all other elements of X and no element of X is a predecessor of a.

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B.Sc - part - II, paper - III, Date - 21-04-2020

Differential Calculus (partial differentiation)

Q.1. If $u = \cos^{-1}\frac{x-y}{x+y}$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

Soln: We have $\cos u = \frac{x-y}{x+y}$

It is clear that $\frac{x-y}{x+y}$ is of zero degree and hence $\cos u$ is a homogeneous function of ~~degree~~ zero degree.

$\therefore$ By Euler's theorem,

$$x\frac{\partial(\cos u)}{\partial x} + y\frac{(\cos u)}{\partial y} = n\cos u = 0\times\cos u = 0$$

$$\Rightarrow -x\sin u.\frac{\partial u}{\partial x} - y\sin u.\frac{\partial u}{\partial y} = 0 \Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$ Proved

Q.2. If $u = \sin^{-1}\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

Soln: We have $\sin u = \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

Since, $\sin u$ is a homogeneous function of zero degree.

$\therefore$ By Euler's theorem,

$$x\frac{\partial(\sin u)}{\partial x} + y\frac{\partial}{\partial y}(\sin u) = 0\times\sin u$$

$$\Rightarrow \cos u\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right) = 0$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

Proved

B.Sc-part-II, paper-III, 21-04-2020

Differential Calculus (partial differentiation)

Q.3. If $u = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$$

Soln: We have, $u = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$

$$\Rightarrow \cos u = \frac{x+y}{\sqrt{x}+\sqrt{y}}$$

$$\Rightarrow \cos u = \frac{x\left(1+\frac{y}{x}\right)}{\sqrt{x}\left(1+\sqrt{\frac{y}{x}}\right)}$$

$$\Rightarrow \cos u = \frac{x^{\frac{1}{2}}\left(1+\frac{y}{x}\right)}{1+\sqrt{\frac{y}{x}}} = v \text{ (say)} \quad \text{—— (I)}$$

We find that v is a homogeneous function of two independent variable x and y and of degree $\frac{1}{2}$. Hence applying Euler's theorem for the function v we have

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = nv = \frac{1}{2}v \quad \text{—— (II)}$$

From I, $v = \cos u$

$$\therefore \frac{\partial v}{\partial x} = -\sin u\frac{\partial u}{\partial x} \text{ and } \frac{\partial v}{\partial y} = -\sin u\frac{\partial u}{\partial y}$$

Substituting these values in II, we get

$$-x\sin u\frac{\partial u}{\partial x} - y\sin u\frac{\partial u}{\partial y} = \frac{1}{2}\cos u$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\cot u$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0.$$

Proved

B.Sc part-III(H), paper-III, 21-04-2020

Differential calculus (partial differentiation)

Q.4. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$.

Soln: $\sin u = \frac{x+y}{\sqrt{x}+\sqrt{y}} = v$ (say)

Since v is a homogeneous function of x and y of degree $\frac{1}{2}$

$\therefore$ By Euler's theorem.

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = nv$$

$$\Rightarrow x\frac{\partial}{\partial x}(\sin u) + y\frac{\partial}{\partial y}(\sin u) = \frac{1}{2}\sin u$$

$$\Rightarrow x\cos u\frac{\partial u}{\partial x} + y\cos u\frac{\partial u}{\partial y} = \frac{1}{2}\sin u$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$$ Proved.

Q.5. If $u = \sin(\sqrt{x}+\sqrt{y})$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}(\sqrt{x}+\sqrt{y})\cot(\sqrt{x}+\sqrt{y})$$

Soln: We have, $u = \sin(\sqrt{x}+\sqrt{y}) \Rightarrow \sin^{-1}u = \sqrt{x}+\sqrt{y}$

$$\Rightarrow \sin^{-1}u = \sqrt{x}\left(1+\sqrt{\frac{y}{x}}\right) = V \text{ (say)} \quad \text{— (I)}$$

We find that V is a homogeneous function of two independent variables x and y of degree $\frac{1}{2}$.

$\therefore$ By Euler's theorem, we get

$$x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} = nV = \frac{1}{2}V \quad \text{— (II)}$$

From (I), $V = \sin^{-1}u$

$$\therefore \frac{\partial V}{\partial x} = \frac{1}{\sqrt{1-u^2}}\frac{\partial u}{\partial x} \text{ and } \frac{\partial V}{\partial y} = \frac{1}{\sqrt{1-u^2}}\frac{\partial u}{\partial y}$$

Substituting these two values in II, we get

$$\frac{x}{\sqrt{1-u^2}}\frac{\partial u}{\partial x} + \frac{y}{\sqrt{1-u^2}}\frac{\partial u}{\partial y} = \frac{1}{2}\sin^{-1}u$$

$$\Rightarrow \frac{1}{\sqrt{1-u^2}}\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right) = \frac{1}{2}(\sqrt{x}+\sqrt{y})$$

Differential Calculus (partial diff). B.Sc. part-II(H), paper-III. 21-04-2020.

Rest part of Q.5.

$$\Rightarrow x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}(\sqrt{x}+\sqrt{y})\sqrt{1-u^2}$$

$$=\frac{1}{2}(\sqrt{x}+\sqrt{y})\sqrt{1-\sin^2(\sqrt{x}+\sqrt{y})}$$

Hence, $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}(\sqrt{x}+\sqrt{y})\cos(\sqrt{x}+\sqrt{y})$ Proved.

Q.6. If $u=\sin^{-1}\frac{x^2+y^2}{x+y}$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$

Soln: We have, $u=\sin^{-1}\frac{x^2+y^2}{x+y}$

$$\Rightarrow \sin u=\frac{x^2+y^2}{x+y}=\frac{x^2\left(1+\frac{y^2}{x^2}\right)}{x\left(1+\frac{y}{x}\right)}=x\phi\left(\frac{y}{x}\right)=v, \text{ say}$$

Now, v is a homogeneous function of degree 1

∴ By Euler's theorem, we get

$$x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}=1.v$$

But $v=\sin u$

$$\therefore \frac{\partial v}{\partial x}=\cos u\frac{\partial u}{\partial x} \text{ and } \frac{\partial v}{\partial y}=\cos u\frac{\partial u}{\partial y}$$

Substituting these values in (I), we get

$$\therefore x\cos u\frac{\partial u}{\partial x}+y\cos u\frac{\partial u}{\partial y}=\sin u$$

$$\Rightarrow x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$$ Proved.

Q.7. If $\sin u=\frac{x^3+y^3}{x-y}$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2\tan u$

Soln: Since, $\sin u=\frac{x^3+y^3}{x-y}$

Here, we see that $\sin u$ is a homogeneous function of the second degree.

∴ By Euler's theorem, we get

$$x\frac{\partial}{\partial x}(\sin u)+y\frac{\partial}{\partial y}(\sin u)=2\sin u$$

$$\Rightarrow x\cos u\frac{\partial u}{\partial x}+y\cos u\frac{\partial u}{\partial y}=2\sin u$$

$$\Rightarrow x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2\tan u$$

Proved